AQA

General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MPC1

Unit Pure Core 1

Monday 11 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 13x - 12$.

(a) Use the Factor Theorem to show that $x + 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(b) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

2 The triangle ABC has vertices $A(1, 3)$, $B(3, 7)$ and $C(-1, 9)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence show that angle ABC is a right angle. *(2 marks)*

(b) (i) Find the coordinates of M, the mid-point of AC. *(2 marks)*

(ii) Show that the lengths of AB and BC are equal. *(3 marks)*

(iii) Hence find an equation of the line of symmetry of the triangle ABC. *(3 marks)*

3 The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(3 marks)*

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. *(4 marks)*

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. *(1 mark)*

4 (a) Show that $\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}$ is an integer and find its value. *(3 marks)*

(b) Express $\dfrac{2\sqrt{7}-1}{2\sqrt{7}+5}$ in the form $m+n\sqrt{7}$, where m and n are integers. *(4 marks)*

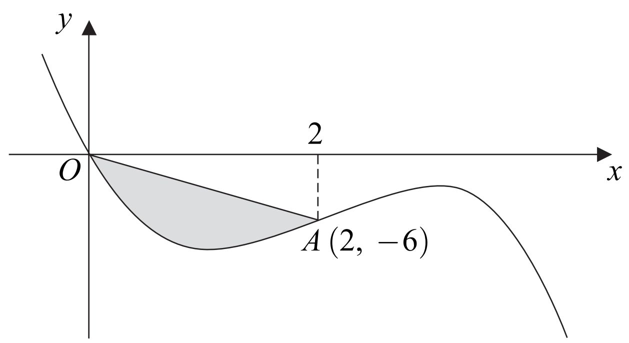
5 (a) Express $(x-5)(x-3)+2$ in the form $(x-p)^2+q$, where p and q are integers. *(3 marks)*

(b) (i) Sketch the graph of $y=(x-5)(x-3)+2$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(ii) Write down an equation of the tangent to the graph of $y=(x-5)(x-3)+2$ at its vertex. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y=x^2$ onto the graph of $y=(x-5)(x-3)+2$. *(3 marks)*

6 The curve with equation $y=12x^2-19x-2x^3$ is sketched below.



The curve crosses the x-axis at the origin O, and the point $A\,(2,\ -6)$ lies on the curve.

(a) (i) Find the gradient of the curve with equation $y=12x^2-19x-2x^3$ at the point A. *(4 marks)*

(ii) Hence find the equation of the normal to the curve at the point A, giving your answer in the form $x+py+q=0$, where p and q are integers. *(3 marks)*

(b) (i) Find the value of $\displaystyle\int_0^2 (12x^2-19x-2x^3)\,\mathrm{d}x$. *(5 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

Turn over for the next question

7 A circle with centre C has equation $x^2 + y^2 - 4x + 12y + 15 = 0$.

(a) Find:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the x-axis. *(2 marks)*

(c) The point P with coordinates $(5, k)$ lies outside the circle.

(i) Show that $PC^2 = k^2 + 12k + 45$. *(2 marks)*

(ii) Hence show that $k^2 + 12k + 20 > 0$. *(1 mark)*

(iii) Find the possible values of k. *(4 marks)*

END OF QUESTIONS